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T.Y.B.Sc. (Sem.-V) Examination October - 2025

Mathematics MTH-504 Real Analysis-II (Old)

[Total Marks: 50

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) All questions are compulsory. (3) Figures to the right indicate marks of the corresponding question.	Student's Signature

Q.1. Answer the following (Any five): **10**

1. If $|x + 1| < \delta$, then show that $|x^3 + 1| < 7\delta$
2. Define continuous function in a metric space.
3. Define open ball in R^1 with illustration.
4. If $\lim_{x \to a} f(x) = L$ and $c \in R$, and then show that $\lim_{x \to a} c \cdot f(x) = cL, c \neq 0$ in a metric space $\langle M, \rho \rangle$.
5. In R_d find an open ball about 'a' with radius $\frac{1}{4}$.
6. Give an example of a Cauchy sequence which is not convergent.
7. If f is a real valued function, then define limit of function f at infinity.

   Show that in any metric $\langle M, \rho \rangle$, the empty set ϕ is open.

Q.2. Answer the following (Any two): **10**

1. If the real valued function f and g are continuous at $a \in R^1$, then prove that f + g and $max\{f, g\}$ are also continuous at $a \in R^1$.
2. Define a function $d: R^2 \to R^+ \ni d(x,y) = |x-y|$ = distance between $x = (x_1, y_1)$ and $y = (x_2, y_2)$ Then show that $\langle R^2, d \rangle$ is a metric space.
3. Let f and g are real valued functions on real line R. If $\lim_{x \to \infty} f(x) = L$ and $\lim_{x \to \infty} g(x) = M$ then show that $\lim_{x \to \infty} (f(x) + g(x)) = L + M$

Q.3. Answer the following (any two) : **10**

1. Show that if $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in R_d, then there exists $N \in I$, such that $x_N = x_{N+1} = x_{N+2} \cdots$.
2. Prove that the metrics τ and σ on R^2 defined as $\tau(P, Q) = |x_1 - x_2| + |y_1 - y_2\backslash$ and $\sigma(P, Q) = max\{|x_1 - x_2|, |y_1 - y|\}$ are equivalent, where $P(x_1, y_1)$, $Q(x_2, y_2) \in R^2$.
3. Show that a sequence in R_d is convergent if and only if all terms of the sequence are same from some point on.

Q.4. **Answer the following (Any two) :** **10**

1. Let $\langle M_1, \delta_1\rangle$, $\langle M_2, \delta_2\rangle$ and $\langle M_3, \delta_3\rangle$ be metric spaces. If $f: M_1 \to M_2$ is continuous at $a \in M_1$ and $g : M_2 \to M_3$ is continuous at $f(a) \in M_2$ then $gof: M_1 \to M_3$ is continuous at $a \in M_1$.
2. Let $\langle M_1, \delta_1\rangle$ and $\langle M_2, \delta_2\rangle$ be metric spaces. Prove that $f: M_1 \to M_2$ is continuous at $a \in M_1$ if and only if for given $\varepsilon > 0$, there exists $\delta > 0$ such that $f^{-1}(B[f(a);\varepsilon]) \supset B[a;\delta]$.
3. Prove that the real valued function f is continuous at $a \in R^1$ if and only if a sequence $\{x_n\}_{n=1}^{\infty}$ of real numbers converges to point $a \in R^1$ then a sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to f(a).

Q.5. **Answer the following (Any two) :** **10**

1. Prove that a finite intersection of open sets in any metric space is open. Is an arbitrary intersection of open sets in any metric space is open? Justify your answer.
2. Let $\langle M_1, \rho_1\rangle$ and $\langle M_2, \rho_2\rangle$ be two metric spaces and let $f : M_1 \to M_2$. If f is continuous on M_1 then show that $f^{-1}(G)$ is open in M_1 whenever G is open in M_2.
3. Define an open set in a metric space. If set A and B are open in R^1, then show that $A \times B$ is open in R^2
